

## The dependence of the number of pomerons on the impact parameter and the long-range rapidity correlations in $pp$ collisions

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The simple model which enables to take into account the effects of a colour string fusion in  $pp$  interactions is suggested. The parameters of the model are connected with the parameters of the pomeron trajectory and its couplings to hadrons. On the base of the model the MC algorithm which enables to calculate the long-range correlation functions between multiplicities and between the average transverse momentum and the multiplicity in  $pp$  collisions is developed.

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## 1. Introduction

The soft part of the multi-particle production in  $pp$  collisions at high energy, which can not be directly described in terms of the perturbative QCD, is usually described in the framework of string models [1]–[4], which originate from the Gribov-Regge approach. In these models at first stage the colour strings are stretched between the projectile and target partons. The formation of the pair of strings corresponds to the cut pomeron in the Gribov-Regge approach. The hadronization of these strings produces the observed hadrons.

In the case of nuclear collisions, the number of strings grows with the growing energy and the number of nucleons of colliding nuclei, and one has to take into account the interaction between strings in the form of their fusion and/or percolation [5]–[7]. The fusion process results in the reduction of total multiplicity of charged particles and growth of transverse momentum, that was confirmed later [8, 9] in comparison with RHIC data.

The possible experimental observation of the string fusion or percolation is extremely interesting. Therefore, the investigation of long-range correlations (LRC) between multiplicities and transverse momenta in two separated rapidity intervals were proposed as the main tool to study this phenomenon [10]–[17].

Up to the present the string fusion model was usually used for describing the multiplicity and mean transverse momentum of charged particles and their correlation only in the case of nucleus-nucleus collisions. However, the experimental data indicates an increase with energy of the mean transverse momentum and its correlation with multiplicity also in  $pp$  collisions [18]–[21]. In present work we formulate the simple model which enables to take into account the effect of colour string fusion on the LRC between multiplicities and transverse momenta in  $pp$  interactions.

## 2. Formulation of the model

To take into account the effect of string fusion in  $pp$  collisions one needs to know the distribution of strings in the transverse plane at given value of the impact parameter  $b$ . We'll do this in an analogy with the case of nucleus-nucleus collisions.

### 2.1 Distribution of strings in the transverse plane: $AA$ interactions

In the case of  $AA$  interactions one usually assumes that at high energy a number of primary formed quark-gluon strings is proportional to the number of binary inelastic nucleon-nucleon interactions in a given event of nucleus-nucleus scattering [2, 12]. In frame of the classical Glauber model mean number of inelastic  $NN$  interactions in  $AB$  scattering at a fixed impact parameter  $b$  is given by the expression (e.g. see [22, 23]):

$$\langle N_{str}(b) \rangle \sim \langle N_{coll}^{in}(b) \rangle = AB \frac{\sigma_{NN}^{in}}{\sigma_{AB}(b)} \int T_A(\vec{s} - \vec{b}/2) T_B(\vec{s} + \vec{b}/2) d^2\vec{s}. \quad (2.1)$$

Here  $\sigma_{NN}^{in}$  – the cross-section of inelastic nucleon-nucleon interaction,  $\sigma_{AB}(b)$  – the probability of interaction of two nuclei at a given impact parameter  $b$  with at least one inelastic  $NN$  interaction. An integral of  $\sigma_{AB}(b)$ :

$$\sigma_{AB} = \int \sigma_{AB}(b) d^2\vec{b} \quad (2.2)$$

gives a so-called «production cross section» of  $AB$  interaction. The  $\langle \dots \rangle$  in equation (2.1) means an averaging over all events with a given  $b$ .  $T_A$  and  $T_B$  – the profile functions of the colliding nuclei:

$$T_A(\vec{s}) = \int_{-\infty}^{+\infty} \rho_A(\vec{s}, z) dz, \quad (2.3)$$

where  $\rho_A(\vec{r})$  – the nucleon density of nucleus  $A$ , normalized to unity,  $\vec{r} = (\vec{s}, z)$ . The  $\vec{s}$  is a two-dimensional vector in the impact parameter plane.

By (2.1) in the nucleus-nucleus collision at the impact parameter  $b$  for the string density in transverse plane at a point  $\vec{s}$  we have:

$$d\langle N_{str}(b) \rangle / d^2\vec{s} \equiv w_{str}(\vec{s}, \vec{b}) \sim w_{coll}^{in}(\vec{s}, \vec{b}) = AB \frac{\sigma_{NN}^{in}}{\sigma_{AB}(b)} T_A(\vec{s} - \vec{b}/2) T_B(\vec{s} + \vec{b}/2) \quad (2.4)$$

Note that in the transverse plane we place the origin in the middle between the centers of colliding nuclei.

## 2.2 Distribution of strings in the transverse plane: $pp$ interactions

By (2.4) it is natural to suppose that in the case of proton-proton collision at the impact parameter  $b$  the string density in transverse plane at a point  $\vec{s}$  is proportional to

$$w_{str}(\vec{s}, \vec{b}) \sim T(\vec{s} - \vec{b}/2) T(\vec{s} + \vec{b}/2) / \sigma_{pp}(b), \quad (2.5)$$

where now the  $T(\vec{s})$  is the partonic profile function of nucleon. The  $\sigma_{pp}(b)$  is the probability of non-diffractive  $pp$  interaction (with at least one cut pomeron) at a given impact parameter  $b$ . An integral of  $\sigma_{pp}(b)$ :

$$\sigma_{pp} = \int \sigma_{pp}(b) d^2\vec{b} \quad (2.6)$$

gives a non-diffractive cross-section of  $pp$  interaction.

By analogy with light nuclei we will use for the partonic profile function of nucleon the simplest gaussian distribution:

$$T(s) = \frac{e^{-s^2/\alpha^2}}{\pi\alpha^2}. \quad (2.7)$$

Substituting (2.7) in (2.5) one gets

$$w_{str}(\vec{s}, \vec{b}) \sim e^{-(\vec{s}+\vec{b}/2)^2/\alpha^2} e^{-(\vec{s}-\vec{b}/2)^2/\alpha^2} / \sigma_{pp}(b) = e^{-2s^2/\alpha^2} e^{-b^2/2\alpha^2} / \sigma_{pp}(b). \quad (2.8)$$

We see that in this approximation the dependencies on  $b$  and  $s$  are factorized and after integration on  $\vec{s}$  one gets for the mean number of strings in the  $pp$  collision at the impact parameter  $b$ :

$$\langle N_{str}(b) \rangle \sim e^{-b^2/2\alpha^2} / \sigma_{pp}(b). \quad (2.9)$$

Since in this approach the formation of each pair of strings corresponds to one cut pomeron,  $N_{str} = 2N$ , where  $N$  is the number of cut pomerons in a given event, hence (2.9) leads to

$$\langle N(b) \rangle \sim e^{-b^2/2\alpha^2} / \sigma_{pp}(b), \quad (2.10)$$

which gives the dependence of the average number of cut pomerons on the impact parameter in non-diffractive  $pp$  collisions.

### 2.3 Event-by-event fluctuations of the number of cut pomerons at given impact parameter

For the calculation of LRC one should know not only the mean number of pomerons in  $pp$  collisions at a given impact parameter  $b$ , but also the event by event distribution of the number of pomerons around this mean value. We assume that this distribution  $\tilde{P}(N, b)$  at a given value of the impact parameter  $b$  at  $N \geq 1$ :

$$\tilde{P}(N, b) = P(N, b) / [1 - P(0, b)] \quad (2.11)$$

is the simple modification of the poissonian distribution:

$$P(N, b) = e^{-\bar{N}(b)} \bar{N}(b)^N / N! \quad (2.12)$$

with some parameter  $\bar{N}(b)$ . The difference of our distribution  $\tilde{P}(N, b)$  (2.11) from the poissonian one (2.12) is only in excluding of the point  $N = 0$  from it:  $\tilde{P}(0, b) = 0$ , which corresponds to the absence of the non-diffractive scattering at  $N = 0$ . Clear that at  $N \geq 1$  this reduces only to the introduction of the additional common normalization factor in (2.11), which enables the proper normalization:  $\sum_{N=1} \tilde{P}(N, b) = 1$ .

The calculation of the mean number of pomerons at a given  $b$  with the distribution (2.11) gives:

$$\langle N(b) \rangle = \bar{N}(b) / [1 - P(0, b)] . \quad (2.13)$$

Since the probability  $\sigma_{pp}(b)$  of the non-diffractive  $pp$  interaction at the given fixed impact parameter  $b$  is equal to the probability to have a nonzero number of cut pomerons, then

$$\sigma_{pp}(b) = 1 - P(0, b) = 1 - \exp(-\bar{N}(b)) . \quad (2.14)$$

Comparing now the formulae (2.10) and (2.13) with taking into account (2.14) we see that in our model  $\bar{N}(b) \sim e^{-b^2/2\alpha^2}$  or introducing a parameter  $N_0$ :

$$\bar{N}(b) = N_0 e^{-b^2/2\alpha^2} \quad (2.15)$$

Then the mean number of cut pomerons at an impact parameter  $b$  is given by

$$\langle N(b) \rangle = \bar{N}(b) / [1 - \exp(-\bar{N}(b))] , \quad (2.16)$$

where the  $\bar{N}(b)$  is given by (2.15).

### 2.4 Integration over impact parameter - min.bias $pp$ collisions

It is convenient to introduce in the impact parameter plane a density of the probability of non-diffractive  $pp$  interaction (see (2.6)) normalized to unity:

$$f(b) = \sigma_{pp}(b) / \sigma_{pp} , \quad \int f(b) d^2\vec{b} = 1 . \quad (2.17)$$

Using the formula (2.17) one can find a mean number of pomerons in non-diffractive  $pp$  interaction averaged over the impact parameter:

$$\langle N \rangle = \int \langle N(b) \rangle f(b) d^2\vec{b} = \int \bar{N}(b) d^2\vec{b} / \sigma_{pp} = 2\pi\alpha^2 N_0 / \sigma_{pp} \quad (2.18)$$

and the corresponding variance  $D_N \equiv \langle N^2 \rangle - \langle N \rangle^2$ , where

$$\langle N^2 \rangle = \int \langle N^2(b) \rangle f(b) d^2 \vec{b} = \pi \alpha^2 N_0 (N_0 + 2) / \sigma_{pp} = \langle N \rangle (N_0 + 2) / 2 \quad (2.19)$$

Here we have used (2.15). In our model using (2.15) we can calculate also the cross-section of non-diffractive  $pp$  interaction

$$\sigma_{pp} = \int \sigma_{pp}(b) d^2 \vec{b} = \int [1 - \exp(-\bar{N}(b))] d^2 \vec{b} = 2\pi \alpha^2 \Phi_1(N_0) , \quad (2.20)$$

where

$$\Phi_1(z) = \int_0^z (1 - e^{-t}) \frac{dt}{t} , \quad \Phi_m(z) = \sum_{k=1}^{\infty} \frac{(-1)^{k+1} z^k}{k! k^m} . \quad (2.21)$$

## 2.5 Probability of $N$ cut pomerons in non-diffractive $pp$ collision

In the framework of our simple model we can find the probability  $w_N$  to have  $N$  cut pomerons in a non-diffractive  $pp$  collision by averaging the  $\tilde{P}(N, b)$  (2.11) over  $b$  at fixed  $N$ :

$$w_N = \int \tilde{P}(N, b) f(b) d^2 \vec{b} = \frac{1}{\sigma_{pp}} \int P(N, b) d^2 \vec{b} , \quad (2.22)$$

where we have taken into account (2.14). Using now (2.12), we have

$$w_N = \frac{1}{\sigma_{pp} N!} \int e^{-\bar{N}(b)} (\bar{N}(b))^N d^2 \vec{b} = \frac{2\pi}{\sigma_{pp} N!} \int_0^{\infty} e^{-\bar{N}(b)} (\bar{N}(b))^N b db . \quad (2.23)$$

One can introduce in (2.23) the new integration variable  $\bar{N}$  in accordance with (2.15):

$$\bar{N} = \bar{N}(b) = N_0 e^{-b^2/2\alpha^2} , \quad d\bar{N} = -(\bar{N}/\alpha^2) b db . \quad (2.24)$$

Then (2.23) takes the following form

$$w_N = \frac{2\pi \alpha^2}{\sigma_{pp} N!} \int_0^{N_0} e^{-\bar{N}} \bar{N}^{N-1} d\bar{N} . \quad (2.25)$$

Such integral is a difference of the gamma and incomplete gamma functions:

$$\int_0^{N_0} e^{-z} z^{N-1} dz = \int_0^{\infty} e^{-z} z^{N-1} dz - \int_{N_0}^{\infty} e^{-z} z^{N-1} dz = \Gamma(N) - \Gamma(N, N_0) , \quad (2.26)$$

At integer  $N$

$$\Gamma(N) = (N-1)! , \quad \Gamma(N, N_0) = (N-1)! e^{-N_0} \sum_{l=0}^{N-1} N_0^l / l! . \quad (2.27)$$

Gathering we find

$$w_N = \frac{2\pi \alpha^2}{\sigma_{pp} N} \left[ 1 - e^{-N_0} \sum_{l=0}^{N-1} N_0^l / l! \right] = \frac{\sigma_N}{\sigma_{pp}} , \quad (2.28)$$

where we have introduced the  $\sigma_N$  by

$$\sigma_N \equiv \frac{2\pi \alpha^2}{N} \left[ 1 - e^{-N_0} \sum_{l=0}^{N-1} N_0^l / l! \right] , \quad (2.29)$$

The direct summing gives

$$\sum_{N=1}^{\infty} \sigma_N = 2\pi \alpha^2 \Phi_1(N_0) = \sigma_{pp} , \quad (2.30)$$

where we have used (2.20) and (2.21). Recall that  $\sigma_{pp}$  is the non-diffractive  $pp$  cross section (2.6).

## 2.6 Comparison with quasi-eikonal and Regge approaches

Now we see that our formula for the  $\sigma_N$  (2.29) coincides with the well known result for the cross-section  $\sigma_N$  of  $N$  cut-pomeron exchange, obtained in the quasi-eikonal and Regge approaches [24, 25, 26] :

$$\sigma_N = \frac{4\pi\lambda}{CN} \left[ 1 - e^{-z} \sum_{k=0}^{N-1} z^k / k! \right], \quad (2.31)$$

where

$$z = \frac{2\gamma C}{\lambda} \exp(\Delta\xi), \quad \lambda = R^2 + \alpha'\xi, \quad \xi = \ln(s/1\text{GeV}^2). \quad (2.32)$$

Here  $\Delta$  and  $\alpha'$  are the residue and the slope of the pomeron trajectory. The parameters  $\gamma$  and  $R$  characterize the coupling of the pomeron trajectory with the initial hadrons. The quasi-eikonal parameter  $C$  is related to the small-mass diffraction dissociation of incoming hadrons.

This enables to connect the parameters  $N_0$  and  $\alpha$  of our model, which describe the dependence of the mean number of pomerons on the impact parameter  $b$  (see formulae (2.15) and (2.16)) with the parameters of the pomeron trajectory and its couplings to hadrons. Comparing (2.29) and (2.31) we have

$$N_0 = z = \frac{2\gamma C}{\lambda} \exp(\Delta\xi), \quad \alpha = \sqrt{\frac{2\lambda}{C}}, \quad \lambda = R^2 + \alpha'\xi \quad (2.33)$$

In our calculations the numerical values of the parameters for the case of  $pp$  collisions were taken from the paper [27]:

$$\Delta = 0.139, \quad \alpha' = 0.21 \text{ GeV}^{-2}, \quad \gamma = 1.77 \text{ GeV}^{-2}, \quad R^2 = 3.18 \text{ GeV}^{-2}, \quad C = 1.5, \quad (2.34)$$

which gives, for example, for the parameters  $N_0$  and  $\alpha$  entering in our formula (2.15):

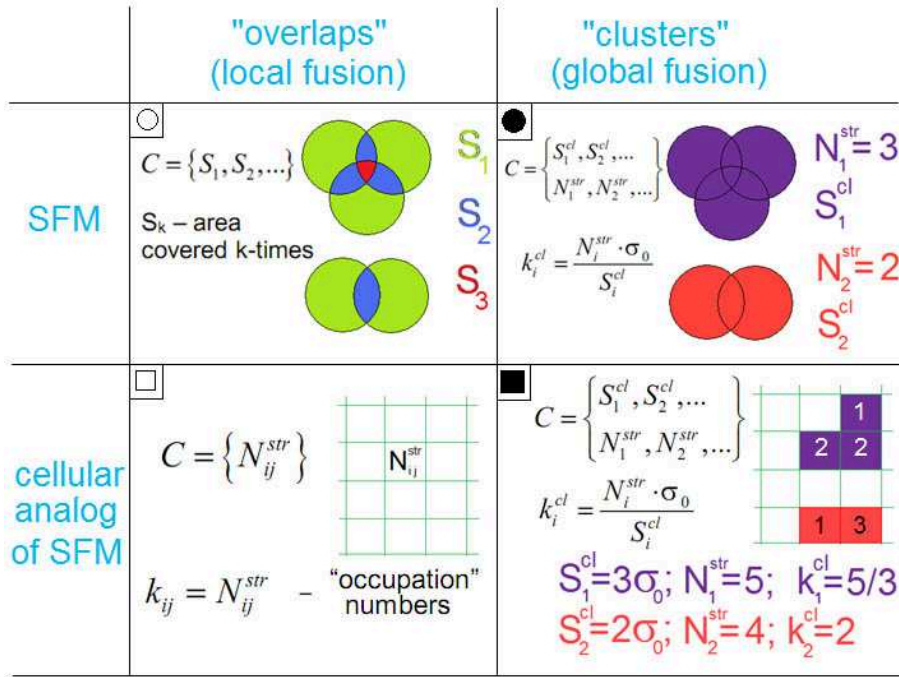
$$\begin{aligned} \alpha &= 0.51\text{fm}, \quad N_0 = 3.38 \quad \text{at } \sqrt{s} = 60\text{GeV}, \\ \alpha &= 0.60\text{fm}, \quad N_0 = 9.02 \quad \text{at } \sqrt{s} = 7000\text{GeV}. \end{aligned} \quad (2.35)$$

## 3. String fusion effects

As follows from the consideration in the previous section, in  $pp$  interactions the number of contributing cut pomerons (see formulae (2.15) and (2.16)) increases with the collision centrality and energy (2.35). In our approach the number of strings is twice the number of cut pomerons, so it also will increase with the collision centrality and energy. Since the strings have a certain limited size in the transverse plane (a plane of the impact parameter) an overlap of the strings will start with increase of their density. As a result the color fields of different strings will interact what will influence on their fragmentation process. For taking into account the effects from the interaction of strings at high density a string fusion model (SFM) was proposed [5]–[7].

### 3.1 Different versions of string fusion model

There are two versions of the SFM a model with local fusion (overlaps model) [28] and a model with formation of global color clusters (clusters model) [29]. In both versions the fusion



**Figure 1:** The different versions of the SFM. The first column corresponds to the version with a local string fusion (overlaps model). The second column corresponds to the formation of fused string clusters (clusters model). The first row corresponds to the original versions of the SFM [28, 29] and the second row corresponds to their cellular analogs [30, 31, 33, 34] (see text for details).

processes result in the reduction of total multiplicity of charged particles and growth of transverse momentum.

In the first variant according to [28] we assume that the mean multiplicity per unit of rapidity and the mean transverse momentum of charged particles emitted from the region, where  $k$  strings are overlapping, are described by the following expressions:

$$\langle n \rangle_k = \mu_0 \sqrt{k} S_k / \sigma_0, \quad \langle p_t^2 \rangle_k = p_0^2 \sqrt{k}, \quad k = 1, 2, 3, \dots \quad (3.1)$$

Here  $S_k$  is the transverse area of the region, where  $k$  color strings are overlapped.  $\sigma_0 = \pi r_{str}^2$  is the transverse area of a string.  $\mu_0$  and  $p_0$  are the mean multiplicity per unit of rapidity and the mean transverse momentum of charged particles, produced from a decay of one string.

In the second variant one assumes that the fused strings form a cluster. According to [29] in this case we suppose that the mean multiplicity per unit of rapidity and the mean transverse momentum of charged particles emitted by a cluster with a transverse area  $S_{cl}$ , formed by  $k$  strings, can be found as follows

$$\langle n \rangle_{cl} = \mu_0 \sqrt{k_{cl}} S_{cl} / \sigma_0, \quad \langle p_t^2 \rangle_{cl} = p_0^2 \sqrt{k_{cl}}, \quad k_{cl} = k \sigma_0 / S_{cl}. \quad (3.2)$$

Note that in two limiting cases (an absence of overlaps of strings and a total overlapping of string areas) both variants lead to the same rules.

Later the simplified discrete analogs of both mentioned versions of SFM based on the implementation of the lattice in the transverse plane [30, 31, 32] were proposed. It was demonstrated



[33, 34] that the MC algorithms of the calculations of LRC coefficients based on the cellular versions of SFM work much faster and give practically the same results as the ones based on the original versions of SFM. These cellular versions of SFM enable also to calculate analytically the mean values of the observables and LRC coefficients in some limiting cases [31, 34], what was used to control the reliability of the created MC codes.

In Fig.1 we illustrate visually the prescriptions of all these versions of the SFM. The first column corresponds to the version with a local string fusion (overlaps model). The second column corresponds to the formation of fused string clusters (clusters model). The first row corresponds to the original versions of the SFM and the second row corresponds to their cellular analogs.

### 3.2 Monte-Carlo algorithm for the calculation of LRC functions

The calculation of LRC functions (regressions) is based on the formula

$$\langle n_B \rangle_{n_F} = \frac{\sum_C w(C) \langle n_B \rangle_C P_C(n_F)}{\sum_C w(C) P_C(n_F)} \quad (3.3)$$

obtained in [13] for the correlation between multiplicities  $n_F$  and  $n_B$  in separated rapidity windows and on the similar formula

$$\langle p_{tB} \rangle_{n_F} = \frac{\sum_C w(C) \langle p_{tB} \rangle_C P_C(n_F)}{\sum_C w(C) P_C(n_F)} \quad (3.4)$$

used in [31, 32, 33] for the calculations of the LRC function between the multiplicity  $n_F$  in the forward rapidity window and the corresponding mean transverse momentum  $p_{tB}$  of  $n_B$  charged particles in the backward window.

The calculations of the sums on string configurations  $C$  in numerator and denominator of the formulae (3.3) and (3.4) were performed by MC simulations of the configurations  $C$  with proper weights  $w(C)$ :

$$\sum_C w(C) \dots = \frac{1}{n_{sim}} \sum_{sim} \dots \quad (3.5)$$

With this purpose at the generation of the string configurations  $C$  the results of the Section 2 were used.

For the generated string configuration  $C$  the mean values of the multiplicity  $\langle n_B \rangle_C$  and transverse momentum  $\langle p_{tB} \rangle_C$  in the backward rapidity window, entering the formulae (3.3) and (3.4), were calculated using the prescriptions (3.1) and (3.2) for the cases of local string fusion and cluster formation, both in frameworks of the original version of SFM and its cellular analog.

We also have supposed that the multiplicity distribution of charged particles produced from the decay of any string is poissonian, what leads to the poissonian distribution for the probability of production of  $n_F$  charged particles  $P_C(n_F)$  from the given string configuration  $C$ :

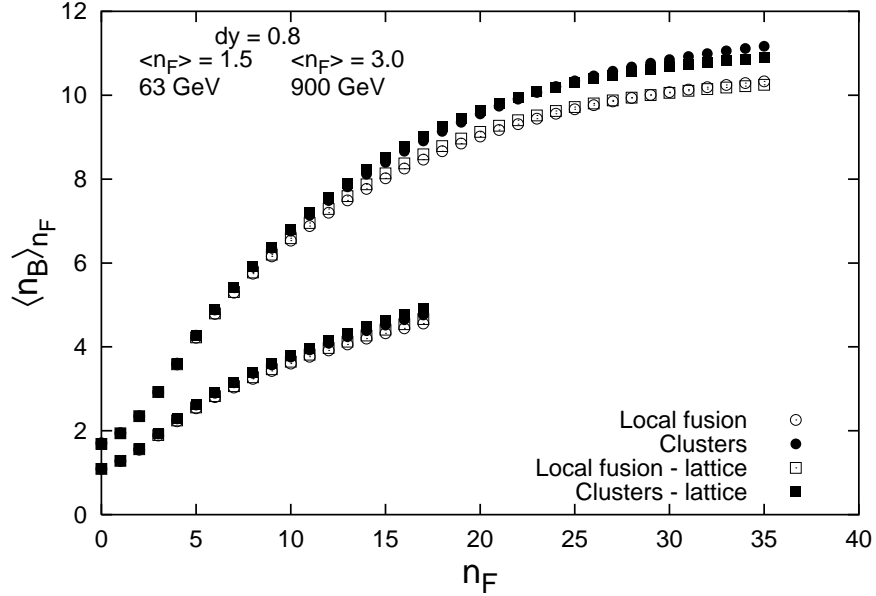
$$P_C(n_F) = P_{\langle n_F \rangle_C}(n_F) = e^{-\langle n_F \rangle_C} \frac{(\langle n_F \rangle_C)^{n_F}}{n_F!}, \quad (3.6)$$

where one can calculate  $\langle n_F \rangle_C$  similar to  $\langle n_B \rangle_C$  using the prescriptions (3.1) and (3.2).

Similarly to the correlation functions (3.3) and (3.4) one can also calculate (by MC simulations of the string configurations  $C$ ) the overall mean values (averaged over all events) in the forward

$$\langle n_F \rangle = \sum_C w(C) \langle n_F \rangle_C, \quad \langle p_{tF} \rangle = \sum_C w(C) \langle p_{tF} \rangle_C \quad (3.7)$$





**Figure 2:** The long-range correlation (LRC) function in  $pp$  collisions between multiplicities  $n_F$  and  $n_B$  in separated rapidity windows calculated by (3.3) at energies 63 and 900 GeV for the forward and backward rapidity windows of the same width  $\Delta y_F = \Delta y_B = dy = 0.8$ , what corresponds to the mean multiplicities  $\langle n_F \rangle = \langle n_B \rangle = \langle n \rangle = 1.5$  and  $3.0$  per window at these energies. Points  $\circ$  and  $\bullet$  - the results of calculations in the framework of the original versions of the string fusion model (SFM) with the local fusion (overlaps) or the formation of string clusters (clusters) correspondingly (see first row in Fig.1) [28, 29]. Points  $\square$  and  $\blacksquare$  - the results of calculations in the framework of the lattice analog of SFM with the local string fusion or the cluster formation correspondingly (see second row in Fig.1) [30, 31, 33, 34].

and (by the same way) in the backward windows.

Note that the resulting distribution on the number of charged particles produced in the forward or backward window

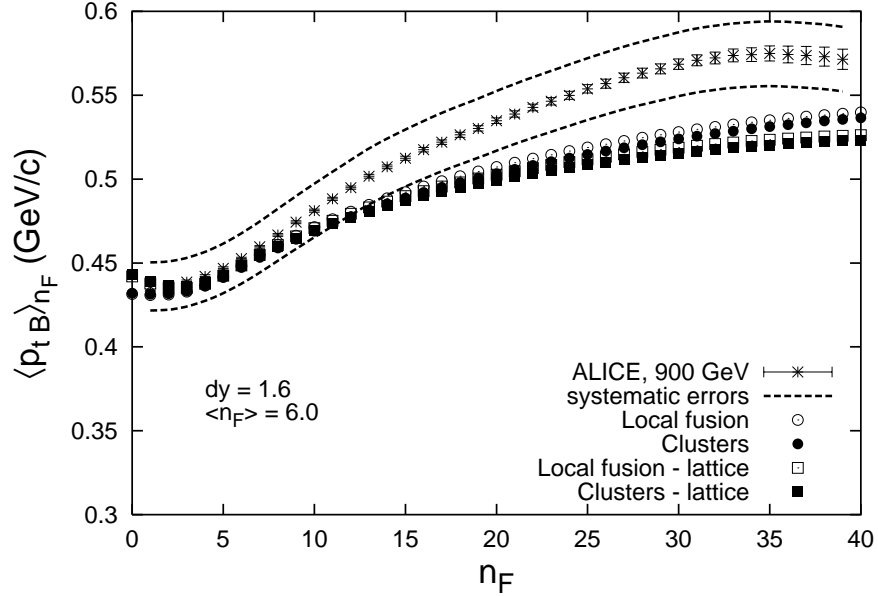
$$P(n_F) = \sum_C w(C) P_C(n_F) \quad (3.8)$$

will be non-poissonian despite the poissonian form (3.6) of the distribution  $P_C(n_F)$  and almost poissonian (see (2.11) and (2.12)) fluctuations in the number of cut pomerons at fixed value of the impact parameter  $b$  in our model. The reason is the non-poissonian fluctuations in the number of cut pomerons originating from the event-by-event fluctuation of the impact parameter  $b$  (see also [33, 35, 36]).

#### 4. Results

In Figs. 2 and 3 we present as an example the results of our calculations of the LRC functions in  $pp$  interactions using the MC algorithm based on the model described above.

In Fig.2 the results of calculations of LRC between multiplicities  $n_F$  and  $n_B$  in separated rapidity windows on the base of the formula (3.3) at the initial energies 63 and 900 GeV are presented. In both cases the rapidity width of the forward and backward windows is taken the same  $\Delta y_F = \Delta y_B = dy = 0.8$ , what corresponds to the mean multiplicities  $\langle n_F \rangle = \langle n_B \rangle = \langle n \rangle = 1.5$  and  $3.0$



**Figure 3:** The long-range correlation (LRC) function in  $pp$  collisions between the multiplicity  $n_F$  in the forward rapidity window and the corresponding mean transverse momentum  $p_{tB}$  of  $n_B$  charged particles in the backward window calculated on the base of the formula (3.4) at the energy 900 GeV for the forward and backward rapidity windows of the same width  $\Delta y_F = \Delta y_B = dy = 1.6$ , what corresponds to the mean multiplicity  $\langle n_F \rangle = \langle n_B \rangle = \langle n \rangle = 6.0$  in the window. Notations of the points are the same as in Fig.2. Points  $\times$  - the experimental data of the ALICE collaboration [37] on the correlation between the multiplicity  $n$  and the mean transverse momentum  $p_t$  of charged particles in the same pseudorapidity interval  $\eta \in (-0.8, 0.8)$ ,  $\Delta\eta = 1.6$  obtained in  $pp$  collisions at 900 GeV.

in such window at these energies. We see in Fig.2 the considerable increase of the strength of the LRC between multiplicities in separated rapidity windows.

In Fig.3 the results on the LRC between the multiplicity  $n_F$  in the forward rapidity window and the corresponding mean transverse momentum  $p_{tB}$  of  $n_B$  charged particles in the backward window on the base of the formula (3.4) at the initial energy 900 GeV are presented. The rapidity width of the forward and backward windows is taken the same  $\Delta y_F = \Delta y_B = \Delta y = 1.6$ , what corresponds to the mean multiplicities  $\langle n_F \rangle = \langle n_B \rangle = \langle n \rangle = 6.0$  in such window at this energy. In Fig.3 we also present the experimental data of the ALICE collaboration [37] on the correlation between the multiplicity  $n$  and the mean transverse momentum  $p_t$  of charged particles in the same pseudorapidity window  $\eta \in (-0.8, 0.8)$ ,  $\Delta\eta = 1.6$  obtained in  $pp$  collisions at the energy 900 GeV. The difference between calculated LRC function and the ALICE experimental data at large multiplicities can be explained by the contributions of additional short-range mechanisms in the case of the correlation between the multiplicity  $n$  and the mean transverse momentum  $p_t$  in the same window, which do not contribute in the case of LRC.

We see also in Figs. 2 and 3 that the results of calculations of the LRC strength in the frameworks of all four versions of SFM (local fusion or cluster formation and their lattice analogs, see Figs.1) turn out to be very close to each other. On the one hand, because of the principal technical differences in the realization of these SFM versions, it indicates in favour of the reliability of the obtained results. On the other hand, it obviously means that one can not distinguish between

versions of the SFM mechanisms on the base of this comparison of the obtained results with the experimental data and the calculations of other string fusion effects on the physical observables are needed.

## 5. Conclusions

The simple model which enables to take into account the effect of colour string fusion in  $pp$  interactions is suggested. At the formulation of the model we assume that the dependence of the average number of cut pomerons on the impact parameter in a non-diffractive (ND)  $pp$  collision is gaussian (2.15) with the additional condition that we have at least one cut pomeron (2.16). We assume also that the event-by-event distribution of the number of cut pomerons around this average value at fixed impact parameter  $b$  is poissonian (2.12) with the same condition (2.11).

It is shown that these two simple assumptions after integration over impact parameter lead to the well known formula for the cross-section  $\sigma_N$  with  $N$  cut-pomeron exchange in a ND  $pp$  collision (2.29), (2.31), which was obtained in the quasi-eikonal and Regge approaches [24, 25, 26]. This have enabled us to connect the parameters of our model with the parameters of the pomeron trajectory and its couplings to hadrons (2.33).

The effects of the string fusion [5, 6] on the multiparticle production were taken into account in the same way as it was done in the case of AA collisions [32, 33]. At that the different version of the string fusion mechanism (local fusion or cluster formation and their lattice analogs, see Fig.1) were considered.

On the base of the model the Monte-Carlo algorithm was developed and the long-range correlation functions between multiplicities and between the average transverse momentum and the multiplicity in  $pp$  collisions at different energies were calculated. It was found that the results of calculations of the long-range correlation (LRC) strength in the frameworks of all four versions of the string fusion model (SFM) turn out to be very close to each other (see Figs. 3.3 and 3.4).

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